ANALYSIS AND TOPOLOGY HOMEWORK 3 (07/19)

(1) Compute the particular solution to

$$x'' + x' + 2x = 2\cos t$$

such that $x(0) = x'(0) = 1$. Write the solution (response) in the format $A\cos(\omega t - \phi)$. Here A is the amplitude, ω is the frequency and ϕ is the **phase lag** of the response with respect to the input $2\cos t$. Draw the graphs of both the input function $2\cos t$ and the response function in the same coordinate system.

(2) (a). Find the particular solution $x_p(t)$, by ERF, to the equation

$$\tilde{x}'' + \frac{1}{100}\tilde{x}' + 100\tilde{x} = \cos(\omega t).$$

(b). Write $\tilde{x}_p(t)$ in the format of a single cosine function as in the previous question. What is its amplitude A? What ω value maximizes the amplitude and what is the maximum value?

(c). When $\omega = 10$, write down $\tilde{x}_p(t)$. Compute $\tilde{x}_p(0)$ and $\tilde{x}_p'(0)$. Then compute the solution $x(t)$ to the equation

$$x'' + 100x = \cos(10t)$$

with the same initial conditions $x(0) = \tilde{x}_p(0)$ and $x'(0) = \tilde{x}_p'(0)$.

(d). Compare $\tilde{x}_p(t)$ and $x(t)$ (draw the graph). This is an example showing how a small damping term $\frac{1}{100}\tilde{x}'$ removes resonance. Explain why by considering what happens when t increases to ∞.

(3) Find the general solution to

$$y'' + 2y' - 3y = e^t \sin t.$$

(4) Find the general solution to

$$y'' + 2y' + y = \cos t + \sin t.$$

(There are at least two ways to solve this problem. Try to think multiple ways!)