

Homework 1/ Algebraic combinatorics I

- (1) Given a group, is the identity element always unique? Is the inverse of an element always unique? Prove your assertions.
- (2) Given a group G and a subgroup H , will the (left) coset space G/H always form a group in the most natural (obvious) way? If not, try to write down some general conditions you need, so that it does form a group under those conditions.
- (3) Given a natural number n , find the number of integer solutions to the equation $x_0 + \dots + x_n = (n + 1)k$, where x_0, \dots, x_n and k are variables, and $0 \leq x_i \leq n$. (The ordering of x_0, \dots, x_n matters.)