LINEAR ALGEBRA HOMEWORK 4

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Exercise 0.1. *WRITE UP by Jul 31 (Transpose) Define the* **transpose** *operation*

$$^T : M_2 \to M_2, \quad \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \mapsto \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}.$$

(a) Show that $(AB)^T = B^T A^T$ *for* $A, B \in M_2$.
(b) Prove that A *is invertible iff its* A^T *is invertible. Moreover, in this case* $(A^T)^{-1} = (A^{-1})^T$.
(c) Prove that A *is invertible iff its rows form a basis of* F^2.
(Hint: Use (a) and the MTC.)
Generalize this to $n \times n$.

Exercise 0.2. *Decide if* $A = [e_3, e_1 + e_2, e_2] \in M_3$ *is invertible. If so, compute* A^{-1}.

Exercise 0.3. *WRITE UP by Jul 31 For* $X \in M_n$, *put* $k(X) := \dim \ker X$. *Assume* $X^2 = 0$. *In class, you saw that* $k(X) \geq n/2$. *In less than 1 page, show that the following:*
(a) A conjugation class $[X]$ *in* $\mathrm{sol}(X^2 = 0)$ *in* M_n *is uniquely determined by* $k(X)$.
(b) Given any integer $k \geq n/2$, *there is a unique conjugation class* $[X]$ *of such solutions such that* $k(X) = k$.
After doing this right, you will be quite close to solving Project 3, Problems 1 and 2.