LINEAR ALGEBRA HOMEWORK 5

LECTURER: BONG H. LIAN

Exercise 0.1. WRITE UP *Let* $p(t) = t^2 - 3t + 1$. *Take* $F = \mathbb{C}$. *Classify the solutions of* $p(x) = 0$ *in* $M_{n\times n}(\mathbb{C})$.

Exercise 0.2. *('Splitting' a map) Given any linear map* $f : V \to W$, *show that there exist subspaces* $V' \subset V$, $W' \subset W$ *such that*

$$f : V = \ker f + V' \to \operatorname{im} f + W'$$

maps $V' \xrightarrow{\cong} \operatorname{im} f$, *and that both sums are independent sums.*

Exercise 0.3. WRITE UP *Find all conjugacy classes of solutions to the matrix equation*

$$X^3 = 0$$

in $M_3(\mathbb{C})$.

Exercise 0.4. WRITE UP *Let* A *be an* F*-algebra and* V *be a finite dimensional* A*-space. Show that* V *is a quotient* A*-space of a direct sum* $A^{\oplus k}$ *of* k *copies of* A*, regarded as an* A*-space. In other words, there exists a surjective* A*-space homomorphism*

$$\pi : A^{\oplus k} \twoheadrightarrow V.$$

We say that an A*-space* M *is semi-minimal if it decomposes into a independent sum of* A*-subspaces which are minimal. Show that if* A *is semi-minimal as an* A*-space, then any* A*-space* V *is semi-minimal.*

Exercise 0.5. *Let* $x, y \in M_{n,n}$. *Recall that* x, y *are conjugates of each other iff there exists an invertible matrix* g *such that* $y = g^{-1}xg$. *Prove your assertions.*
(a) Suppose $\det x \neq \det y$. *Can* x, y *be conjugates of each other, i.e. can* $[x] = [y]$*?*
(b) Suppose $\det x = \det y$. *Does this imply that* $[x] = [y]$*?*