LINEAR ALGEBRA HOMEWORK 6

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Exercise 0.1. *Decide if* $A = [e_3, e_1 + e_2, e_2] \in M_3$ *is invertible. If so, compute* A^{-1}.

Exercise 0.2. *You will prove that there is a bijection between the set of conjugation classes of* $n \times n$ F*-matrices, and the set of isomorphism classes of* $F[t]$*-spaces* V *of* $dim_F V = n$. *To each matrix* x, *define the* $F[t]$*-space by the* F*-algebra homomorphism*

$$\varphi_x : F[t] \to \mathrm{End}\, F^n \equiv M_{n\times n}, \quad t \mapsto x.$$

Argue if $h \in \mathrm{Aut}_n$, *then* $\varphi_{h^{-1}xh}$ *defines an isomorphic* $F[t]$*-space. Verify that the correspondence* $[x] \mapsto [\varphi_x]$ *is a bijection from conjugation classes of matrices to isomorphism classes of* $F[t]$*-spaces.*

Exercise 0.3. *WRITE UP For* $X \in M_n$, *put* $k(X) := \dim\ker X$. *Assume* $X^2 = 0$.
(a) Show that $k(X) \geq n/2$.
In less than 1 page, show that the following:
(a) A conjugation class $[X]$ *in* $\mathrm{sol}(X^2 = 0)$ *in* M_n *is uniquely determined by* $k(X)$.
(b) Given any integer $k \geq n/2$, *there is a unique conjugation class* $[X]$ *of such solutions such that* $k(X) = k$.
After doing this right, you will be quite close to solving Project 3, Problems 1 and 2.

Exercise 0.4. *WRITE UP Let* $C_\bullet$ *be a complex of* F*-spaces with* $C_0 = F^2$, $C_1 = F^3$, *and* $C_j = 0$ *for all* $j \neq 0, 1$. *Decide which of the following homology of* $C_\bullet$ *is possible:*
(a) $H_0 = F$, $H_1 = F^2$.
(b) $H_0 = F^2$, $H_1 = F^3$.
(c) $H_0 = F$, $H_1 = F$.
(d) $H_0 = 0$, $H_1 = F^2$.