

## Homework 2/ Algebraic combinatorics

- (1) Write down the definition of a subgroup of a group.
- (2) Let  $H$  be a subgroup of  $G$ . Prove that  $G/H$  is a group under the induced group operation from  $G$ , if and only if  $H$  is a normal subgroup of  $G$ .
- (3) Let  $\phi : G \rightarrow H$  be a group homomorphism. Prove that
  - (a)  $Im(\phi)$ , the image of  $\phi$ , is a subgroup of  $H$ .
  - (b)  $Ker(\phi)$ , the kernel of  $\phi$ , is a normal subgroup of  $G$ .
  - (c)  $G/Ker(\phi) \cong Im(\phi)$  as groups.