



## ANALYSIS AND NUMBER THEORY, YAU TSINGHUA MATHCAMP, SUMMER 2023

### COURSE SYLLABUS

#### 1. GENERAL INFORMATION

**Instructor:** Cezar Lupu (BIMSA)

**Coaches:** Dongsheng Wu (BIMSA), Tianyu Zhang (Ohio State University)

**Email addresses:** lupucezar@gmail.com (Cezar Lupu), wudongsheng14@mails.ucas.ac.cn (Dongsheng Wu), bidenbaka@gmail.com (Tianyu Zhang)

**Lecture & Tutorial session times:**

- Monday-Friday (Lecture), 13:30-15:00 PM
- Monday-Friday (Tutorial session), 15:30-17:30 PM & 19:00-20:00 PM

**Location:** A120, Tsinghua International Mathematics Forum (TSIMF)

**Office hours (1 on 1 meetings with coaches):** Make an appointment with the coaches via email

- Dongsheng Wu
- Tianyu Zhang

**Topics covered:** sequences and series of real numbers, sequences and series of real functions, mean value theorems for differentiable functions and Riemann integrals, integral inequalities (Cauchy-Schwarz, Holder, Jensen etc), convex functions, arithmetic functions, asymptotic formulas for arithmetic functions, partial summation, Euler-Maclaurin summation, prime numbers, prime counting function, Bertrand's postulate, Riemann zeta function for real arguments and its special values.

#### 2. COURSE DESCRIPTION

In this course, we explore the connections between two important fields of mathematics such as analysis and number theory. First, we introduce the necessary analytic tools such as series of real numbers, derivatives and integrals, sequences and series of functions. Moreover, we study arithmetic functions, but from an analytic perspective, dwelling more on their asymptotic expansion. Also, we study the distribution of prime numbers via elementary methods. One of the key figures for this endeavor is the celebrated Riemann zeta function (for real argument) which can be regarded as an important tool in connection with prime numbers.

By the end of this course, students should develop fundamental knowledge and skills involving basic concepts in real analysis and number theory. This course will serve as an essential ingredient for an introductory analytic number theory course in college.

Last but not least, students will have the opportunity to present research projects on topics related to the content of this course.

## Course Outline

- **Chapter 0.** What is the connection between analysis and number theory? The big picture.
- **Chapter 1.** Infinite sequences and series of real numbers. Convergence and examples. Convergence criteria.
- **Chapter 2.** Derivatives and integrals of functions on the real line and their properties.
- **Chapter 3.** Sequences and series of real numbers. Pointwise and uniform convergence.
- **Chapter 4.** Asymptotics and summation formulas.
- **Chapter 5.** Arithmetic functions. Elementary properties and asymptotic estimates.
- **Chapter 6.** Prime numbers and their properties and distribution. Elementary properties.
- **Chapter 7.** The Riemann zeta function for real argument and its special values.

### 3. GRADING POLICY, GRADING SCALE, WEIGHTED VALUE OF ASSIGNMENTS AND TESTS

- Homework assignments: **100%** (4 homework assignments in total; you must solve the assigned problems from each homework to get full credit!). There will be around 40 problems in total. Each homework will consist of 10 problems. However, the coaches will **grade randomly only 5 problems for each assignment.**

Letter grades will then be assigned in accordance with the following correspondence:

- Letter **grade A** = a percentile grade of 90% or higher
- Letter **grade B** = a percentile grade of 80% or higher, that is lower than 90%
- Letter **C** = a percentile grade of 70% or higher, that is lower than 80%
- Letter **D** = a percentile grade of 60% or higher lower than 70%
- Letter **F** = a percentile grade lower than 60%