LINEAR ALGEBRA HOMEWORK

JULY 22, 2023

Exercise 1. *Let* $f : A \to B,\ g : B \to C$ *be surjective maps. Show that* $g \circ f$ *is also surjective.*

Exercise 2. *Let* $f : A \to B$ *be a map with inverse* f^{-1}*, then we have* $B \xrightarrow{f^{-1}} A \xrightarrow{f} B$*. Show that* $f \circ f^{-1}$ *is* id_B*.*

Exercise 3. *Let* $f : A \to B$ *be a map with inverse* f^{-1}*. Show that* f^{-1} *is a bijection.*

Exercise 4. *Let* $f : A \to B,\ g : B \to A$ *be two maps. Assume* $gf = id_A,\ fg = id_B$*.*

(1) *Show that* f *is a bijection.*
(2) *Show that the inverse of* f *is* g*.*

Exercise 5.

(1) *If* F *is a field such that* $|F|$ *is a prime number* p*, show that* F *is a unique isomorphism to a* $\mathbb{Z}/p\mathbb{Z} = \{\bar{0}, \bar{1}, \ldots, \overline{p-1}\}$*.*
(2) *If* F *is a field such that* $|F|$ *is finite, then* $|F| = p^r$*, where* p *is a prime,* $r \in \mathbb{Z}_+$*. (Hint: Assume that* $2 = 0$ *in* F*. Define that the map* $\mathbb{Z}/2 \to F$*, assigning 0,1 in* $\mathbb{Z}/2$ *to 0,1 in* F*. Show that this map is linear over the field* $\mathbb{Z}/2$*. [You will have to guess what this means.] Next, consider all possible linear maps* $(\mathbb{Z}/2)^k \to F$ *that is injective for each* $k = 1, 2, \ldots$ *Show that there is a positive integer* r *such that such an injective linear map exists, for* $k = 1, 2, \ldots, r$*. But such an injective linear map does not exist for* $k = r + 1$*.)*