LINEAR ALGEBRA HOMEWORK

JULY 22, 2023

Exercise 1. *Let $f : F^n \to F^m$ have linear properties. That is,*

$$\lambda x \mapsto f(\lambda x) = \lambda f(x), \quad \forall \lambda \in F, \forall x \in F^n;$$

$$x + x' \mapsto f(x + x') = f(x) + f(x'), \quad \forall x, x' \in F^n.$$

Does there exist an $m \times n$ matrix A, such that $f(x) = Ax$?

Exercise 2. *You have now shown that linear maps $F^n \to F^m$ correspond to $m \times n$ matrices bijectively (from the uniqueness and existence results this morning). Consider composing two linear maps $f : F^n \to F^m$, $g : F^m \to F^k$.*

1. *Show that their composition gf is linear.*
2. *Let A, B, C be the matrices corresponding to the linear maps f, g, gf. Write a formula for the coefficients c_{ij} of C, in terms of the coefficients of A, B.*

The matrix C is called the matrix product of A with B. We write $C = BA$.

3. *Consider 3 matrices P, Q, R and assume that the matrix products $(PQ)R$ is defined. Show that $P(QR)$ is also defined, and that the two products are equal.*