LINEAR ALGEBRA HOMEWORK

JULY 25, 2023

In today's homework, F is a field.

Exercise 1. *Assume $v \in F^2$ is a nonzero vector. Let $V = Fv = \{\mu v : \mu \in F\}$ be a subspace of F^2. Prove that*

$$\begin{aligned} \varphi : & F \longrightarrow V \\ & \lambda \longmapsto \lambda v \end{aligned}$$

is a linear bijection, that is, φ is a bijection such that

$$\varphi(\lambda_1 + \mu\lambda_2) = \varphi(\lambda_1) + \mu\varphi(\lambda_2)$$

for any $\mu \in F$ and $\lambda_1, \lambda_2 \in F$.

Exercise 2. *Let V be a subspace of F^n. Let $f : F^k \longrightarrow V$ and $g : F^\ell \longrightarrow V$ be two linear bijections. Show that f^{-1} and $f^{-1}g$ are also linear bijections.*

Exercise 3. *A homogeneous linear system with more variables than equations must have a non-zero solution. Prove this in half a page.*