Problem 1

Damped Oscillator. Consider a (1-dimensional) harmonic oscillator subjected to a force that depend on the velocity of the particle. If we write the position of the particle as a function $x : \mathbb{R} \to \mathbb{R}$, that is, a map $t \mapsto x(t)$, the equation of motion looks like

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

where m is the mass of the particle.

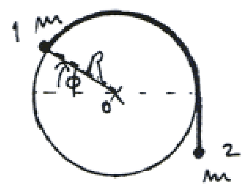
- What are the units of the constants b and k?
- Find the form of most general solution to this differential equation. For this, first show that if $x_1(t)$ and $x_2(t)$ are solutions of this equation, so is $x_1(t) + x_2(t)$. Then, use the fact that the most general class of solutions can be written in the form $Ae^{\lambda t}$ with A and λ constants. Can you determine A and or λ?
- The solution found above has different qualitative behavior depending on the values parameters m, b, k. Explain this.
- Consider the cases found in the previous example and assume we have the initial conditions
$$x(0) = x_0 \qquad \frac{dx(0)}{dt} = \dot{x}_0$$
for some real numbers x_0 $\dot{x}_0$ and then draw the trajectories of the solutions in the phase space (x, p) (where $p = m\frac{dx}{dt}$ is the momentum of x).**Hint**: if it this is hard for you to visualize this, you can take specific values for the constanst $m, b, k, x_0, \dot{x}_0$ and/or use the help of a software of your choice to draw the plots.

Problem 2*

Consider a man walking his dog. The man and the dog's position can be model by a point in $\mathbb{R}^2$. The man moves with constant velocity $\vec{v}_m$ and the dog moves with constant speed $\|\vec{v}_d\|$ and it always moves towards the man. Find an equation for the curve described by the dog (i.e. no need to find $\vec{x}_d(t)$, we want to just know the shape of the curve, not how it evolvs in time, for example describe it as points $(x_1(x_2), x_2) \in \mathbb{R}^2$).

Problem 3

Consider two particles of mass m each joined by a string of mass 0 which cannot stretch (an 'ideal string') of length πR and the system is over a cylinder of radius R (the cylinder is fixed) and at $t = 0$ the particles are located in a symmetric position with respect to the center of the cylinder. giving a very small velocity to the particle on the right so the system star falling by the effect of gravity (pointing towards $-e_3$), as the picture shows.



1. Write the equation of motion of both particles.
2. Write a differential equation of the form $\frac{d^2\phi}{dt^2} = F(\phi)$ for the angle ϕ shown in the picture and integrate it to find an expression for $\dot{\phi} := \frac{d\phi}{dt}$. **Hint**: use the fact that we can write

$$\frac{d^2\phi}{dt^2} = \frac{d\dot{\phi}}{d\phi}\dot{\phi}$$

   and so the equation becomes

$$\dot{\phi}d\dot{\phi} = F(\phi)d\phi$$

   which can be integrated.
3. Find the angle ϕ_0 for which the particle on the left falls off from the cylinder. **Hint**: if a particle have no contact with the cylinder, what happens to the normal force?