ACI HW2 Due 8/8 in class

August 1, 2024

Problem 1. *Draw the Hasse diagram for the intersection poset formed by the line arrangement* $x = 0, x = y, x = -y, x = -1$ *in* $\mathbb{R}^2$*. Compute the Möbius function and characteristic polynomial. Check the Zaslavsky theorem in this case.*

Problem 2. *Let* $\mathcal{H}$ *be an arrangement of hyperplanes in* $\mathbb{R}^n$ *defined by* $x_1 = x_2, x_2 = x_3, \cdots, x_{n-1} = x_n, x_n = x_1$*. Compute the characteristic polynomial* $\chi_{\mathcal{H}}(t)$ *and the number of regions for* $\mathcal{H}$*.*

Problem 3. *Let* $\mathcal{H}$ *be an arrangement of* m *hyperplanes in general position in* $\mathbb{R}^3$*. Compute the characteristic polynomial. (Optional: Compute the characteristic polynomial for the hyperplane arrangements in the general position in* $\mathbb{R}^d$*)*

Problem 4. *Let* G *be a graph with d vertices, and for every pair of vertex, there is an edge. This is called a complete graph. Compute the chromatic polynomial of* G *and the number of acyclic regions in* G*.*

Problem 5. *Let* G *be a graph with* d *vertices. Suppose that* G *has a* k*-element clique, i.e.,* k *vertices such that each pair of vertices is connected by an edge.*

1. *Let* K_k *be the complete graph on* k *vertices. Show that* $\chi_G(t) = \chi_{K_k}(t)f(t)$ *for some polynomial* $f(t)$ *with integer coefficients.*
2. *Show that* $k!$ *divides the number of acyclic orientations of* G*.*

Problem 6. *Let* $\mathcal{H}$ *be a hyperplane arrangement. Show that the coefficients of the characteristic polynomial* $\chi_{\mathcal{H}}$ *have alternating signs.*

Problem 7. *Assume that the hyperplane arrangement* $\mathcal{H}$ *is central. Prove that* $t = 1$ *is a root of* $\chi_{\mathcal{H}}(t)$*.*

Problem 8 (Optional)**.** *Let* $\mathcal{H}$ *be an arrangement of hypersurfaces in* $\mathbb{R}^d$ *defined by* $\vec{n}_i \cdot \vec{x} = b_i$*,* $1 \le i \le m$*. Let* $\widetilde{\mathcal{H}}$ *be the* $m + 1$ *hyperplanes in* $\mathbb{R}^{d+1} = (x_1, \cdots, x_d, y)$ *defined by* $\vec{a}_i \cdot \vec{x} = b_i y$*,* $1 \le i \le m$ *and* $y = 0$*. Prove* $\chi_{\widetilde{\mathcal{H}}} = (t-1)\chi_{\mathcal{H}}$*.*