ACI HW3 Due 8/13 in class

August 8, 2024

Problem 1. *Consider the hyperplane arrangements in* $\mathbb{R}^d$ *defined by* $x_i + x_j = 0$ *and* $x_i - x_j = 0$ *for all pairs* $i \neq j$*. Find the characteristic polynomial.*

Problem 2. *Let* $G = (V, E)$ *be a graph with* d *vertices* $V = \{1, 2, \cdots, d\}$ *and* $\mathcal{H}$ *be the corresponding graphic arrangement in* $\mathbb{R}^d$*. Let* $\mathcal{A}$ *be the set of hyperplanes* $x_i = 0$ *for all* $1 \leq i \leq d$*. Express the characteristic polynomial of* $\mathcal{H} \cup \mathcal{A}$ *in terms of the characteristic polynomial of* $\mathcal{H}$*.*

Problem 3. *Let* ΔABC *be an equilateral triangle on* $\mathbb{R}^2$*, try to find all the reflections in the reflection group generated by the three reflections* R_{AB}, R_{AC}, R_{BC}*.*

Problem 4. *Consider all the reflection symmetries of a cube in* $\mathbb{R}^3$*. The mirrors of these reflections form a hyperplane arrangement. Find the number of regions cut out by this hyperplane arrangement and the order of the reflection group generated by these reflections. (Optional: Find the characteristic polynomial for this arrangement.)*

Problem 5 (optional)**.** *Let* P *be the poset of positive integers defined by* $a \leq b$ *iff* $a|b$*. Prove that* $\mu(a, b) = (-1)^k$ *if* $\frac{b}{a} = p_1 p_2 p_3 \cdots p_k$ *with distinct primes* $p_1, p_2, p_3, \cdots, p_k$*; and* $\mu(a, b) = 0$ *otherwise. Hint: use the following theorem*

Theorem 1. *Let* L *be a finite lattice with at least 2 elements and* $\hat{0} \neq a \in L$*, then*

$$\sum_{a \vee x = \hat{1}} \mu(x) = 0.$$