ALGEBRA AND TOPOLOGY
HOMEWORK FOUR
DUE: 8/5

There are two different types of labels: alphabets and numbers. You only need to write up your solutions to those exercises labelled by alphabets. The rest is for fun.

Exercise A. How is the connected sum of a torus and a Klein bottle classified? What about the connected sum of two Klein bottles?

Exercise B. Compute all the $\mathbf{F}_2$ homology groups for $\mathbf{RP}^2$ and S^2. You have to specify which combinatorial structure (complex) is used in your computation.

Exercise 1. Show that a sequentially compact surface is connected if and only if triangles in a triangulation can be arranged in a sequence $T_1, \dots, T_r$ so that each triangle has at least one edge identified with an edge of an earlier triangle in the sequence.

Exercise 2. State and prove a classification theorem for compact surfaces (not necessarily connected).

Exercise 3. Let F be a field and S be a set. Show that the set of all functions from S to F

$$F^S := \{f \colon S \to F\}$$

is a vector space. You can think of elements in F^S as a sequence indexed by S. In which case, the addition and scalar multiplication on F^S are defined as

$$\{x_s\}_{s\in S} + \{y_s\}_{s\in S} := \{x_s + y_s\}_{s\in S} \text{ and } \lambda \cdot \{x_s\}_{s\in S} := \{\lambda \cdot x_s\}_{s\in S}.$$

Exercise 4. Prove the Claim stated in class: there exists a toroidal pair $\{c, c\}$ that separates the given one $\{a, a\}$.