LINEAR ALGEBRA HOMEWORK 5

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Exercise 0.1. *By row reduction, compute*

$$\det[e_3 + e_2 + e_1, e_1 + e_2, e_2].$$

Redo this it by using linearity of det *in each column.*

Exercise 0.2. *Assume that* $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \in M_{2\times 2}$ *is invertible. Find a formula for* A^{-1}*. That is to say, find each entry of* A^{-1} *in terms of the 4 entries* a_{ij} *of* A*. Be sure to check that you do get* $AA^{-1} = A^{-1}A = I$*. From this, can you guess the answer for* 3×3 *matrices?*

Exercise 0.3. *WRITE UP Prove that the minimal polynomial of a matrix* $A \in M_{n,n}$ *is conjugation invariant, i.e.* $\mu_{g^{-1}Ag}(x) = \mu_A(x)$ *for all* $g \in \mathrm{Aut}_n$*. Conclude that the algebra* $F[x]/\mu_A(x)F[x]$ *does not change under conjugations of* A*.*

Exercise 0.4. *Compute the* $\mu_A(x)$ *for* $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$.

Exercise 0.5. *WRITE UP For a given* $A \in M_{m\times n}$*, propose an algorithm to compute a basis of* $\ker A$ *and* $\mathrm{im}\, A$ *by row operations. In 10 lines prove that your algorithm is correct.*