LINEAR ALGEBRA HOMEWORK 1

LECTURER: BONG H. LIAN

Exercise 0.1. *WRITE UP by July 19, 2019 Recall that for $a, b \in \mathbb{Z}$, $a \neq 0$, the fraction b/a is defined to be the set of all equations*

$$nx = m, \quad n, m \in \mathbb{Z}, \quad n \neq 0, \ s.t. \ nb = ma.$$

Prove that two such sets $b/a = b'/a'$ iff $ba' = b'a$.
Prove that $b/a \cap b'/a' = \emptyset$ iff $ba' \neq b'a$.
Prove that the map $\mathbb{Z} \to \mathbb{Q}$, $n \mapsto n/1$ is injective. Therefore, we can treat $\mathbb{Z}$ is a subset of $\mathbb{Q}$ by treating set $n/1$ as the integer n.
Prove that this map is not surjective.

Exercise 0.2. *WRITE UP by July 19, 2019 Verify that the multiplication rule given by*

$$\times : \mathbb{Q}^2 \to \mathbb{Q}, \ (b/a, b'/a') \mapsto b/a \times b'/a' := (bb')/(aa')$$

is well-defined. Note that this rule generalizes the usual rule for multiplying integers, i.e. grouping apples. Therefore, the multiplication rule for $\mathbb{Z}$ remains the same after we treat it as a subset of $\mathbb{Q}$. (See next exercise.)
Prove using this multiplication rule, that b/a is a solution to the equation $ax = b$. That is, $a \times b/a = b$.)
Prove that this is the only solution. That is, if b'/a' is another solution, then $b'/a' = b/a$.

Exercise 0.3. *Write For $n \in \mathbb{Z}$, put $\iota(n) = n/1$. Verify the identities*

$$\iota(1) = 1/1, \quad \iota(nn') = \iota(n)\iota(n'), \quad n, n' \in \mathbb{Z}.$$

Also express $\iota(n + n')$ in terms of $\iota(n)$, $\iota(n')$.

Exercise 0.4. *Let F be a field. For $X, Y, Z \in F^2$ and $\lambda, \mu \in F$, verify that*
V1. $(X + Y) + Z = X + (Y + Z)$
V2. $X + Y = Y + X$
V3. $X + 0 = X$
V4. $X + (-X) = 0$
V5. $\lambda(X + Y) = \lambda X + \lambda Y$
V6. $(\lambda + \mu)X = \lambda X + \mu X$
V7. $(\lambda\mu)X = \lambda(\mu X)$
V8. $1X = X$.
The same holds true for F^n.
Suggestion: *Think about exactly what facts about F you need to use to prove each of these statements.*