From classical to quantum mechanics

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The discovery of quantum mechanics was, perhaps, the most revolutionary change ever made in our understanding of the physical world. It provides the basis for our current understanding of the structure of atoms and molecules, as well as the properties of materials and light, which are essential to almost every aspect of the technological world in which we now live.

Along with its relativistic version, quantum field theory, quantum mechanics describes all the behaviour of the subatomic world which we can observe today: in a single century, we went from having no good model of the structure of atoms, to being able to calculate their workings with incredible precision.

The invention of quantum mechanics would have been impossible without surprising experimental results of the 19th and 20th centuries. At the same time, it was also founded on successive mathematical formulations of mechanics from the 18th and 19th centuries, by mathematicians including Lagrange and Hamilton.

In this course, we will start by introducing Lagrange and Hamilton's approaches to classical mechanics. The position $r(t)$ and velocity $\dot{r}(t)$ of a particle in the original Newtonian formulation of classical mechanics are determined as a function of time t by Newton's laws of motion. In contrast, the Lagrangian and Hamiltonian formulations treat $r(t)$ and $\dot{r}(t)$ as functions minimizing a functional obtained from an object $\mathcal{L}(r, \dot{r}, t)$ known as the Lagrangian.

This formulation turns out to be very beautiful, but is more than just an aesthetic improvement: we will be able to indicate its connections with very deep branches of mathematics, including functional analysis and symplectic geometry. In the second part of the course, we will see how it opens the door to naturally make the transition to quantum mechanics. One of the main features of the quantum world is that position $r(t)$ and momentum $p \sim \dot{r}(t)$ become independent noncommuting variables $\hat{r}, \hat{p}$ satifying an algebraic relation

$$[\hat{r}, \hat{p}] = i\hbar$$

where $\hbar$ denotes the Planck constant. We will study the consequences of this, and the mathematics related to quantum mechanics, including Hilbert spaces and the algebra of operators related to physical observables. We will then apply this to understand simple quantum systems such as oscillators and atoms, and explore the the consequences of 'quantization' of classical systems, as well as some inherently quantum systems without a classical analogue.